حلول بعض الاسئلة المختارة من شابتر 9

ملاحظة: جميع اسئلة هاذ الشابتر مهمة ولكن لا نستطيع تصوير الحل لجميع الاسئلة.

والحل عبارة عن صور وليس نص مكتوب. واذا لم يوجد رقم السؤال فوق الصورة، فتكون الصورة(الحل) مكملة للصورة(للحل) السابقة.

على سبيل المثال سؤال رقم 14 من سكشن 1، له صورتان

9.1

9.1.3

Step 1 of 2

The sample space S contains 52 cards

$N(S) = 52$

Let E be the event that there is a red card, but not including face cards

The face cards are kings, queens and jacks

Among the red cards there are 2 kings, 2 queens, and 2 jacks

$E = \{2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8, 9, 9, 10, 10,$

$A, A\}$

$\Rightarrow N(E) = 20$

Comment

Step 2 of 2

Thus, the probability that a chosen card is a red card but not a face card is

$$P(E) = \frac{N(E)}{N(S)}$$

$$= \frac{20}{52}$$

$$= \frac{5}{13}$$

9.1

9.1.4

'

Let S be the sample space containing the 52 cards of a pack of cards

Then $N(S)=52$

Let E be the event containing black cards which do not have even numbers on them

The black cards contain 13 spades and 13 clubs

The even numbered cards are 2, 4, 6, 8, and 10 within the 13 spades and 13 clubs

Thus, out of the total 26 black cards there are 10 even numbered cards

E = {2, 2, 4, 4, 6, 6, 8, 8, 10, 10}

$\Rightarrow N(E)=10$

Comment

Step 2 of 2

∴ The probability that a black card is chosen and has an even number on it is

$$P(E)=\frac{N(E)}{N(S)}$$

$$=\frac{10}{52}$$

$$=\frac{5}{26}$$

9.1

9.1.5

Step 1 of 1

Let S be the sample space containing the 52 cards of a pack of cards

Then, $N(S) = 52$

Let E be the event that a 10, jack, queen, king, or an ace is drawn. These cards can be hearts, diamonds, clubs, and spades.

E = { 10, 10, 10, 10, J, J, J, J, Q, Q, Q, Q, K, K, K, K,

A, A, A, A}.

Thus, $N(E) = 20$

$\therefore$ The probability that the chosen card is at least 10 is

$$P(E) = \frac{N(E)}{N(S)}$$

$$= \frac{20}{52}$$

$$= \frac{5}{13}$$

9.1

9.1.6

Consider the sample space:

There are 52 cards in a deck.

The red suits are diamonds and hearts and the black suits are clubs and spades.

Each suit contains 13 cards.

Let S be the sample space containing the 52 cards in a deck.

Then, the total number of cards $N(S)=52$.

Consider the event that the denomination of the chosen card is at most 4.

Let E be the event that the denomination of a chosen card is at most 4.

Then, *E* contains the cards whose denominations are 4, 3, and 2 and of hearts, diamonds, clubs, and spades.

$$E=\{4,4,4,4,3,3,3,3,2,2,2,2\}$$

The number of successful cards $N(E)=12$.

Comment

Step 2 of 2

Objective is to compute the probability of the event that the denomination of the chosen card is at most 4.

Therefore, the probability that the denomination of a chosen card is at most 4 is:

$$P(E)=\frac{N(E)}{N(S)}$$

$$=\frac{12}{52}$$

$$=\boxed{\frac{3}{13}}.$$

Hence, the required probability is $P(E)=\boxed{\frac{3}{13}}$.

9.1

9.1.7

Step 1 of 1

Let S be the sample space containing the set of all outcomes when two dice are rolled

Then,

$$S=\left\{\begin{matrix} 11,12,13,14,15,16 \\ 21,22,23,24,25,26 \\ 31,32,33,34,35,36 \\ 41,42,43,44,45,46 \\ 51,52,53,54,55,56 \\ 61,62,63,64,65,66 \end{matrix}\right\}$$

Then, $N(S)=36$

Let E be the event that the sum of the numbers showing face up equals 8

Then, $E=\{26,62,35,53,44\}$

$\Rightarrow N(E)=5$

$\therefore$ The probability that the sum of the numbers showing face up is 8 is

$$P(E)=\frac{N(E)}{N(S)}$$

$$=\frac{5}{36}$$

9.1

9.1.8

Step 1 of 1

Let S be the sample space containing all the outcomes when two dice are rolled

Then,

$$S = \begin{Bmatrix} 11,12,13,14,15,16 \\ 21,22,23,24,25,26 \\ 31,32,33,34,35,36 \\ 41,42,43,44,45,46 \\ 51,52,53,54,55,56 \\ 61,62,63,64,65,66 \end{Bmatrix}$$

Then, $N(S) = 36$

Let E be the event that the sum of the numbers showing face up are the same

Then, $E = \{11,22,33,44,55,66\}$

$$\Rightarrow N(E) = 6$$

Thus, the probability that the sum of the numbers show the same number is

$$P(E) = \frac{N(E)}{N(S)}$$

$$= \frac{6}{36}$$

$$= \frac{1}{6}$$

9.1

9.1.9

Step 1 of 1

Let S be the sample space containing all the outcomes when two dice are rolled

Then,

$$S = \begin{Bmatrix} 11,12,13,14,15,16 \\ 21,22,23,24,25,26 \\ 31,32,33,34,35,36 \\ 41,42,43,44,45,46 \\ 51,52,53,54,55,56 \\ 61,62,63,64,65,66 \end{Bmatrix}$$

Then, $N(S) = 36$

Let E be the event that the sum of the numbers showing face up is at most 6

$$E = \{15,51,24,42,33,14,41,23,32,13,31,22,11,12,21\}$$

$$\Rightarrow N(E) = 15$$

$\therefore$ The probability that the sum of the numbers showing face up is at most 6 is

$$P(E) = \frac{N(E)}{N(S)}$$

$$= \frac{15}{36}$$

9.1

9.1.10

Step 1 of 1

Let S be the sample space containing all the possible outcomes when two dice are rolled

$$S = \begin{Bmatrix} 11,12,13,14,15,16 \\ 21,22,23,24,25,26 \\ 31,32,33,34,35,36 \\ 41,42,43,44,45,46 \\ 51,52,53,54,55,56 \\ 61,62,63,64,65,66 \end{Bmatrix}$$

Then, $N(S) = 36$

Let E be the event that the sum of the numbers showing face-up is at least 9

$$E = \{36,45,46,54,55,56,63,64,65,66\}$$

$$N(E) = 10$$

$\therefore$ The probability that the sum of the numbers showing face-up is at least 9 is

$$P(E) = \frac{N(E)}{N(S)}$$

$$= \frac{10}{36}$$

$$= \frac{5}{}$$

9.1

9.1.14

(a)

In this problem we should find the probability that exactly one of the three people becomes ill. So, either *A* or *B* or *C* should get illness.

In the first case the person *A* got illness and persons *B* and *C* are not.

The probability of the person *A* to become ill is $\frac{1}{2}$ and the probabilities of the other two persons B and C not becoming ill are $\frac{1}{2}$ and $\frac{1}{2}$ respectively.

Hence by multiplication theorem the probability that A become ill and the other two are not is given by $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{8}$

For the second case replace the person *A* with *B* and the other two persons with *A* and *C*.

The probability of the person B to become ill is $\frac{1}{2}$ and the probabilities of the other two persons A and C not becoming ill are $\frac{1}{2}$ and $\frac{1}{2}$ respectively.

Hence by multiplication theorem the probability that B become ill and the other two are not is given by $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{8}$

For the last case replace the person B with C and the other two persons with *A* and *B*.

The probability of the person C to become ill is $\frac{1}{2}$ and the probabilities of the other two persons A and B not becoming ill are $\frac{1}{2}$ and $\frac{1}{2}$ respectively.

Hence by multiplication theorem the probability that C become ill and the other two are not is given by $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{8}$

So, the probability that exactly one of the people becomes ill $=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$

$=\boxed{\frac{3}{8}}$

9.1

Step 3 of 4

(b)

In this problem we should find the probability that at least two of the people become ill. So either A and B or B and C or C and A should get illness or all the three people get illness.

Let us start with A and B. The probability of A to become ill is $\frac{1}{2}$ and of B is $\frac{1}{2}$.

So, the probability of A and B to become ill is $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{4}$.

Similarly, the probability of B and C to become ill is $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{4}$ and the probability of A and C to become ill is $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{4}$.

So, the probability that at least two of the people become ill $=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$

$$=\frac{3}{4}$$

On the other hand, the probability that all the people get illness $=\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$.

$$=\frac{1}{8}$$

Hence the probability that at least two of the people become ill $=\frac{3}{4}+\frac{1}{8}$

$$=\frac{6+1}{8}$$

$$=\boxed{\frac{7}{8}}$$

9.1

Step 4 of 4

(c)

In this problem we should find the probability that none of the three people becomes ill.

The probability that the person *A* has no illness = $\frac{1}{2}$

The probability that the person *B* has no illness = $\frac{1}{2}$

The probability that the person *C* has no illness = $\frac{1}{2}$

So, the probability that none of the three people becomes ill $= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \boxed{\frac{1}{8}}$

9.1

9.1.19

(a) The total possible outcomes are

$$S=\begin{Bmatrix} B_1B_1, B_1B_2, B_1W_1, B_1W_2, B_1W_3, B_2B_1, B_2B_2, B_2W_1, B_2W_2, B_2W_3, W_1W_1, W_1W_2, \\ W_1W_3, W_1B_1, W_1B_2, W_2W_1, W_2W_2, W_2W_3, W_2B_1, W_2B_2, W_3W_1, W_3W_2, W_3W_3, W_3B_1, \\ W_3B_2 \end{Bmatrix}$$

$$\Rightarrow N(S)=25$$

Comment

Step 3 of 4

(b) The set of all outcomes in which the first ball that is drawn is blue is

$$E_1=\{B_1B_1,\ B_1B_2, B_1W_1, B_1W_2,\ B_1W_3,\ B_2B_1,\ B_2B_2,\ B_2W_1,\ B_2W_2,\ B_2W_3\}$$
$$\Rightarrow N(E_1)=10$$

The probability that the first ball that is drawn is blue is

$$P(E_1)=\frac{N(E_1)}{N(S)}$$

$$=\frac{10}{25}$$
$$=\frac{2}{5}$$

Comment

Step 4 of 4

(c) If only white balls are drawn, then the set of all outcomes is

$$E_2=\{W_1W_1,\ W_1W_2,\ W_1W_3,\ W_2W_1,\ W_2W_2,\ W_2W_3,\ W_3W_1,\ W_3W_2,\ W_3W_3\}$$

The probability that both the balls drawn are white is

$$P(E_2)=\frac{N(E_2)}{N(S)}=\frac{9}{25}$$

9.1

9.1.21

(a) Let the two digit numbers are

$10, 11, 12, \ldots 99$

Now we have to count the number of integers which are multiples of 3.

$$\begin{array}{cccccc} 10, & 11, & 12, 13, 14, 15\ldots & & 96, 97, 98, 99 & \\ & & \updownarrow & \updownarrow \cdots \updownarrow & & \updownarrow \\ & & 3\cdot 4 & 3\cdot 5 \quad 3\cdot 32 & & 3\cdot 33 \end{array}$$

There are $33-4+1=30$ positive two digit integers which are multiple of 3.

Comment

Step 2 of 4

(b)

The total number of two digit integers is

$$\begin{aligned} N(S) &= 99-10+1 \\ &= 90 \end{aligned}$$

And the favorable integers are

$N(E)=30$ (Positive two digit integer which are multiple of 3)

So the required probability

$$\begin{aligned} P(E) &= \frac{N(E)}{N(S)} \\ &= \frac{30}{90} \\ &= \frac{1}{3} \end{aligned}$$

9.1

(c)

Let the two digit numbers are

$10, 11, 12, \dots 99$

Now we have to count the number of integers which are multiples of 4.

$$\begin{array}{llllll} 10, & 11, & 12, 13, 14, 15, & 16\dots & & 96, 97, 98, 99 \\ & & \updownarrow & \updownarrow & \dots & \updownarrow \\ & & 4\cdot 3 & 4\cdot 4 & \dots & 4\cdot 24 \end{array}$$

There are $24-3+1=22$ positive two digit integers which are multiple of 4.

Comment

Step 4 of 4

The total number of two digit integers is

$$\begin{aligned} N(S) &= 99-10+1 \\ &= 90 \end{aligned}$$

And the favorable integers are

$N(E)=22$ (Positive two digit integer which are multiple of 4)

So the required probability

$$\begin{aligned} P(E) &= \frac{N(E)}{N(S)} \\ &= \frac{22}{90} \\ &= \frac{11}{45} \end{aligned}$$

9.1

9.1.23

Step 2 of 6

(a) The number of elements in the array is $n-1+1=n$

Using the result of the theorem that if *m* and *n* are integers, and $m \le n$, then there are $n-m+1$ integers from *m* to *n* inclusive.

Comment

Step 3 of 6

(b) The number of elements in the subarray $A[4], A[5] ...... A[39]$ is

$39-4+1=36$

Comment

Step 4 of 6

(c) Let *S* be the ple space containing the 1-dimensional array $A[1],\ A[2], ... A[n] \quad n \ge 50$

$N(S)=n$

Let *E* be the event containing the elements of the subarray $A[3], A[4], A[5], ... A[m]$

Then, $N(E)=m-3+1$

Thus, the probability that a randomly chosen array element is in the subarray $A[3] ..... A[m]$ is

$$P(E)=\frac{N(E)}{N(S)}$$

$$=\frac{m-3+1}{n}$$

$$=\frac{m-2}{n}$$

9.1

Step 5 of 6

(d) If $n = 39$, then $A[\lfloor n/2 \rfloor] = A[39/2] = A[19]$

Then, the subarray $A[\lfloor n/2 \rfloor], A[\lfloor n/2 \rfloor + 1], \ldots\ldots\ldots A[n]$ reduces to

$A[19], A[20], A[21] \ldots\ldots\ldots A[39]$.

The number of elements is 39 – 19 + 1 = 21

Comment

Step 6 of 6

The array $A[1], A[2], \ldots\ldots\ldots\ldots A[39]$ contains the subarray $A[19] \ldots\ldots\ldots\ldots A[39]$

∴ The probability that a randomly chosen element belongs to the subarray

$$A\lfloor 19 \rfloor \ldots\ldots\ldots\ldots A[39] = \frac{\text{the number of elements in the subarray}}{\text{the number of elements in the array}}$$

$$= \frac{21}{39}$$

9.1

9.1.25

(a) We have to find the number of elements in the subarray $A[\lfloor n/2 \rfloor], A[\lfloor n/2 \rfloor + 1] \ldots\ldots\ldots A[n]$

(i) If n is even, $A[\lfloor n/2 \rfloor] = A[\lfloor n/2 \rfloor]$

Then, the number of elements from $A[\lfloor n/2 \rfloor], A[\lfloor n/2 \rfloor + 1], \ldots\ldots\ldots\ldots A[n]$ is

$= n - (n/2) + 1$

$= (n/2) + 1$

(ii) If n is odd, then $A[\lfloor n/2 \rfloor] = A[(n-1)/2]$

Then, the number of elements from $A[\lfloor n/2 \rfloor], A[[\lfloor n/2 \rfloor] + 1 \ldots\ldots\ldots\ldots A[n]]$ is

$= n - ((n-1)/2) + 1$

$= (2n - n + 1 + 2)/2$

$= (n+3)/2$

Comment

Step **3** of 3

(b) The probability that a randomly chosen array element is in the subarray $A[\lfloor n/2 \rfloor] \ldots\ldots\ldots A[n]$

$$= \frac{\text{the number of elements in the subarray}}{\text{the number of elements in the array}}$$

(i) If n is even, the number of elements in the subarray is $(n/2) + 1$

$\therefore$ The required probability is $\dfrac{(n/2)+1}{n}$

(ii) if n is odd, the number of elements in the subarray is $(n+3)/2$

$\therefore$ The required probability is $= \dfrac{(n+3)/2}{n}$

$$= \frac{(n+3)}{2n}$$

9.1

9.1.30

Step 1 of 3

Consider the integers from 1 to 1001.

The objective is to find how many even integers are there between 1 and 1,001.

Comment

Step 2 of 3

The number of even integers from 1 to 1001

1 2 3 4 ....100 ....1000 1001

$2\times 1 \quad 2\times 2 \quad 2\times 50 \quad 2\times 500$

An even integer is an integer that is exactly divisible by 2, as we can see from the above table.

Comment

Step 3 of 3

The multiples of 2 are $2\times 1, 2\times 2, ...2\times 500$

Therefore, the number of multiples of 2 are the number of integers from 1 to 500 is, $500-1+1$ which is equal to 500.

Thus, there are 500 even integers from 1 to 1001.

Comment

9.1

9.1.33

Step 1 of 4

Let us consider the following statement $P(n)$:

If *m* and *n* are integers and $m \le n$, then there are $n-m+1$ integers from *m* to *n* inclusive.

Comment

Step 2 of 4

Let us prove that $P(1)$ is true.

Since, $n=1$, let us select the value of *m* which is less than or equal to *n*.

Let $m=-1$

Now, from the formula, there must be $1-(-1)+1=3$ integers between 1 and 1 (including 1 and 1).

And from the number line, it is observed that there are 3 integers 1, 0, and 1 between 1 and 1.

Thus, $P(1)$ is true.

9.1

Step 3 of 4

Now, prove that for all integers *k*, if $P(k)$ is true then $P(k+1)$ is also true.

Let us consider the inductive hypothesis $P(k)$

Again, consider $m=-1$`

Now, from the formula, there must be $k-(-1)+1=k+2$ integers between 1 and *k*

Comment

Step 4 of 4

Now, consider the statement $P(k+1)$

Again, consider $m=-1$

Now, if the total integers between 1 and *k* are $k+2$, total integers between 1 and $k+1$ are $k+3$,

Now, from the formula, total number of integers between 1 and $k+1$ must be $(k+1)-(-1)+1=k+3$

Thus, $P(k+1)$ is true using the inductive hypothesis

Therefore, $P(n)$ is true for all integers.

9.2

9.2.12

Step 1 of 2

(a)

Find the number of hexadecimal numbers which begin with any of the digits 3 through *B*.

From 3 to *B*, there are 9 possibilities for the first place.

Similarly, the last place should be filled with any of the digits from 5 through *F*.

From 5 to *F*, there are 11 hexadecimal digits.

This implies that there are 11 possibilities for the last place.

Therefore, the 5-length hexadecimal numbers satisfying the given conditions are

$9\times16\times16\times16\times11=\boxed{405504}$

Comment

Step 2 of 2

(b)

The 6-length hexadecimal numbers which begin with digits 4 through *D*, and end with 2 through *E* have to be found out.

The number of hexadecimal digits from 4 through *D* is 10

The number of hexadecimal digits from 2 to *E* is 13.

The first and last places can be filled in 10 ways and 14 ways, respectively, and the remaining 4 places can be filled in 16 ways each.

Use the rule of rule of multiplication.

Therefore, the total possibilities are $10\times16\times16\times16\times16\times13=\boxed{8519680}$

9.2

9.2.13

(a) When a coin is tossed, the outcomes are either heads or a tails.

When a coin is tossed 4 times, the total possible outcomes are $2\times2\times2\times2=16$ by the rule of multiplication.

$$S=\begin{Bmatrix} HHHH, HHHT, HTHH, THHH, HHTT, HTHT, THHT, \\ HTTH, TTHH, THTH, TTTH, TTHT, THTT, HTTT, TTTT, HHTH \end{Bmatrix}$$

Comment

Step **2** of 3 ^

(b) Let *S* be the sample space containing all the possible outcomes when 4 coins are tossed.

Then, $N(S)=16$

Let E_1 be the event of occurrence of exactly 2 heads.

$$E_1=\{HHTT, HTHT, THHT, HTTH, TTHH, THTH\}$$

Then, $N(E_1)=6$

Then, the probability that exactly two heads occurs is $P(E_1)=\dfrac{N(E_1)}{N(S)}$

$$=\frac{6}{16}$$

$$=\frac{3}{8}$$

9.2

(c) Let S be the sample space containing all the possible outcomes when 4 coins are tossed.

Then, $N(S) = 2^4$

= 16

Let E_2 be the event of obtaining exactly one head.

$$E_2 = \{TTTH, TTHT, THTT, HTTT\}$$

Then, $N(E_2) = 4$

Then, the probability of the occurrence of exactly one head is

$$P(E_2) = \frac{N(E_2)}{N(S)}$$

$$= \frac{4}{16}$$

$$= \frac{1}{4}$$

9.2

9.2.14

(a)

The objective is to determine the number of possible license plates.

Form the license plate as a seven step operation to fill all the symbols.

As there are 26 letters and 10 digits and repetition is allowed, so letters have 26 choices and digits have 10 choices.

There are 26 choices for first symbols, 26 choices for second symbol, 26 symbols for third symbol, 26 symbols for fourth symbol, 10 choices for fifth symbol, 10 choices for sixth symbol and 10 choices for seventh symbol.

By multiplication rule the number of license plates is:

$$26 \times 26 \times 26 \times 26 \times 10 \times 10 \times 10 = 456,976,000.$$

Therefore, there are $\boxed{456,976,000}$ possible number of number plates with four letters and three digits.

Comment

Step **2** of 5

(b)

The objective is to determine the number of license plates that begin with A and end with 0.

As the first symbol is A so the first place has 1 choice.

Second place have 26 choices, 26 symbols for third symbol, 26 symbols for fourth symbol, 10 choices for fifth symbol, 10 choices for sixth symbol.

Final place has 1 choice as the license plate ends with 0.

By multiplication rule the number of license plates is:

$$1 \times 26 \times 26 \times 26 \times 10 \times 10 \times 1 = 1,757,600$$

Therefore, there are $\boxed{1,757,600}$ number plates that begin with A and end with 0.

9.2

(c)

The objective is to determine the number of license plates that begin with $TGIF$.

As the first four symbols are $TGIF$ so the first four places has 1 choice.

Fifth place has10 choices, 10 choices for sixth symbol, and 10 choices for seventh place.

By multiplication rule the number of license plates is:

$$1\times 10\times 10\times 10 = 1,000$$

Therefore, there are $\boxed{1,000}$ number plates that begin with $TGIF$.

Comment

Step 4 of 5

(d)

The objective is to determine the number of license plates if repetition is not allowed.

First place has 26 choices, second place has 25 choices, third place has 24 choices, fourth place has 23 choices, fifth place has 10 choices, sixth place has 9 choices, and seventh place has 8 choices.

By multiplication rule the number of license plates is:

$$26\times 25\times 24\times 23\times 10\times 9\times 8 = 258,336,000$$

Therefore, there are $\boxed{258,336,000}$ number plates if repetition is not allowed.

Comment

Step 5 of 5

(e)

The objective is to determine the number of license plates that begin with AB and all letters and digits are distinct.

First two places are fixed with AB so there is only 1 choice each.

Third place has 24 choices as letters A and B are fixed for first two positions and repetition is not allowed, fourth place has 23 choices, fifth place has 10 choices, sixth place has 9 choices, and seventh place has 8 choices.

By multiplication rule the number of license plates is:

$$1\times 1\times 24\times 23\times 10\times 9\times 8 = 397,440$$

Therefore, there are $\boxed{397,440}$ number plates that begin with AB and all letters and digits are distinct.

9.2

9.2.15

Step 1 of 2

(a) The number of different combinations of locks which require 3 selections of numbers from 1 to 30 is $30 \times 30 \times 30 = 27{,}000$ because there is no restriction on the choice of the numbers.

Comment

Step 2 of 2

(b) The number of different combinations, if no number may be used twice, is $30 \times 29 \times 28 = 24{,}360.$

9.2

9.2.18

(a) From the keypad shown, we understand that the button containing 2 also contains the letters A, B, and C.

Thus, the number of different pins, which represent the same sequence of keys as 2133, can be found by selecting any one from the key containing 2, A, B, and C. This can be done in 4 ways.

Similarly, the key containing 1 also contains Q and Z , so we can select from the 1, Q, and Z in 3 ways.

In the same manner, the key containing 3 also contains D, E, and F, so we can select 4×4 ways for the last 2 places occupying 3 and 3.

Finally, the total number of different pins is $4 \times 3 \times 4 \times 4 = 192$.

Comment

Step 2 of 3

(b) According to the keypad, the key containing 5 also contains J, K, and L, so we can select the first place in 4 ways.

The key containing 0 does not contain any other values, so 0 can be selected in one way.

Similarly, for the third place, there are 4 ways to select to from 3, D, E, and F. Finally, for 1, there are 3 ways to select, i.e., from 1, Q, Z.

Thus, the number of different pins, representing the same sequence of keys as 5031, are $4 \times 1 \times 4 \times 3 = 48$

Comment

Step 3 of 3

(c) The number of numeric sequences which contain no repeated digits is all of the possible arrangements of the 4 digits from 0 to 9. Here, 0 can also occur in the first place, as we are not talking about a 4-digit number.

∴ The required number of numeric sequences that contain no repeated digits is P(10, 4)

$$P(10,4) = \frac{10!}{(10-4)!}$$

$$= \frac{10!}{6!}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!}$$

$$= 10 \times 9 \times 8 \times 7$$

= 5040

9.2

9.2.19

Step 1 of 6

From four people Ann, Bob, Cyd and Dan, 3 of them have to be selected for the positions of president, treasurer and secretary.

However, the restrictions are that Bob is not qualified to be treasurer, and Cyd cannot be the secretary.

Thus, for the post of secretary either Ann, Bob, or Dan can be selected.

Let Ann be selected for the post of secretary.

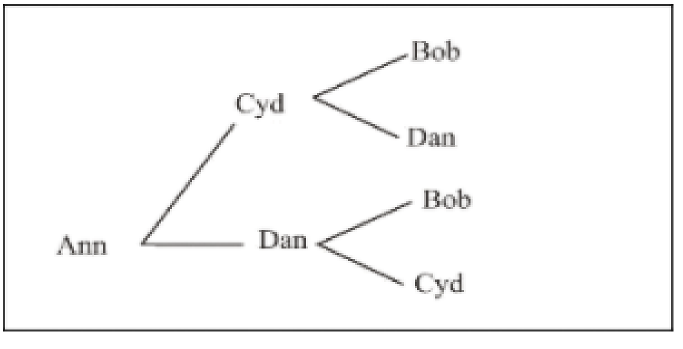
Then, for the remaining two posts, we have to select from Bob, Cyd and Dan.

Therefore, Cyd or Dan can be selected for this post once the treasurer is selected. From the remaining two people, one of them has to be selected for the position of president.

Comment

Step 2 of 6

The following tree diagram shows the Ann is selected for the post of secretary:



Comment

Step 3 of 6

Secretary Treasurer President

Ann Cyd Bob

Ann Cyd Dan

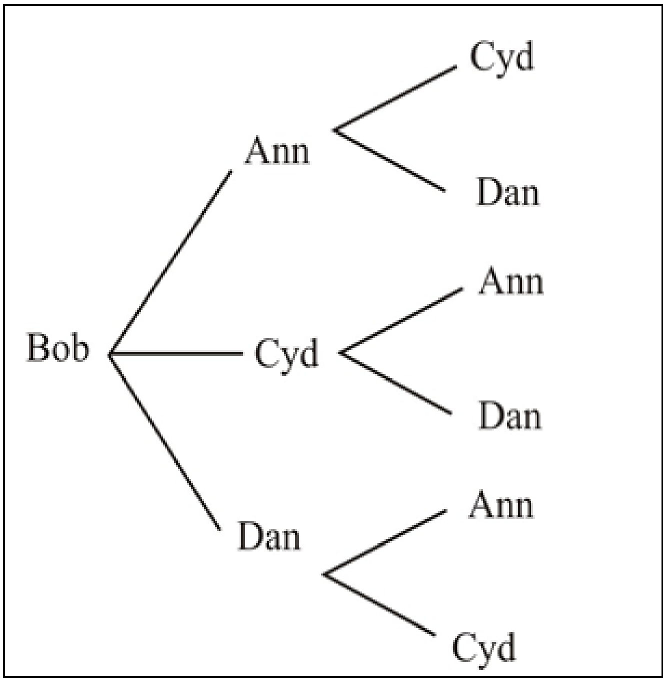
Ann Dan Bob

Ann Dan Cyd

If Bob is selected as secretary, any of Ann, Cyd or Dan can be selected as treasurer.

9.2

The following tree diagram shows the Bob is selected for the post of secretary:



Comment

Step 5 of 6

Secretary Treasurer President

Bob Ann Cyd

Bob Ann Dan

Bob Cyd Ann

Bob Cyd Dan

Bob Dan Ann

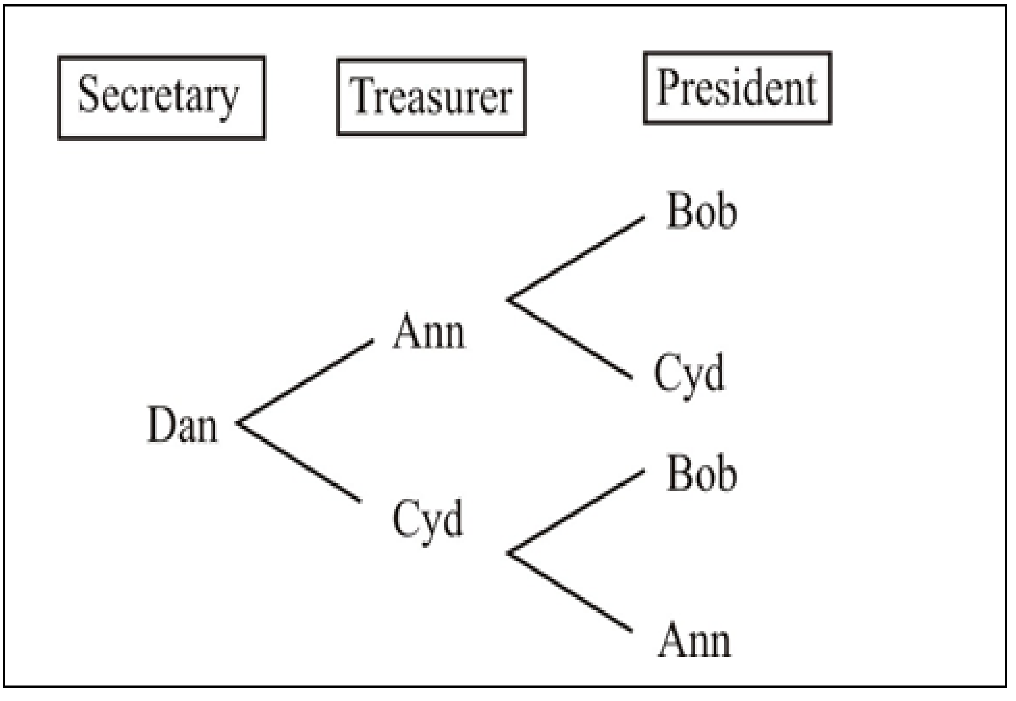
Bob Dan Cyd

If Dan is selected for the position of secretary, then either Ann or Cyd can be selected for the post of treasurer, as Bob is not qualified to be treasurer. Then, any of the remaining 2 people can be selected for the post of president, as there is no restriction on this post.

9.2

Step 6 of 6

The following tree diagram shows the Dan is selected for the post of secretary:



Therefore, the total number of selections is $4+6+4=14$ ways.

The rule of multiplication cannot be used to solve this problem.

9.2

9.2.22

Step 1 of 1

Decimal Interchange Code is a code in which each symbol has an 8-bit representation.

An 8-bit representation of each symbol is unique and is a string of 0's and 1's consisting of 8 locations.

Two choices, 0 or 1, are available for each position in the 8-bit representation.

The objective is to find the number of distinct symbols that can be represented by the Extended Binary Coded Decimal Interchange Code.

By Multiplication Rule, the number of different strings in 8-bit representation is $2^8 = 256$.
Therefore, the number of distinct symbols that can be represented by the code (EBCDIC) is $\boxed{256}$.

9.2

9.2.33

(a) It is given that six people attend the theater together and sit in a row with exactly six Sets.

The number of ways in which they can be seated together in a row is as follows:

$6 \times 5 \times 4 \times 3 \times 2 \times 1$

= 720

Comment

Step 2 of 4

(b) If the doctor is seated in the aisle seat, the remaining 5 people can be seated in the remaining 5 seats in 5! ways

The total possibilities are as follows:

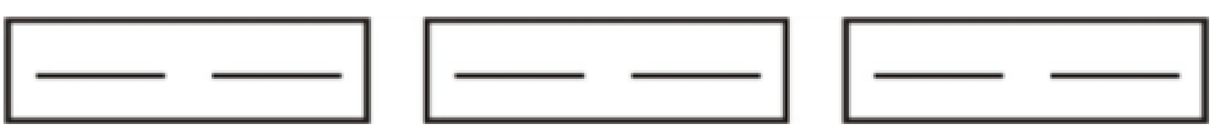
$5 \times 4 \times 3 \times 2 \times 1$

= 120

Comment

Step 3 of 4

(c) Consider that the six people are 3 married couples.



Comment

Step 4 of 4

In continuation to the above, block the six seats into 3 sets of 2 seats each.

In each block, there is one possibility for the husband to be seated on the left and the wife to be seated on the right, and the 3 couples can be seated in the 3 blocks in 3! ways.

The required answer is 6.

9.2

9.2.36

Step 1 of 1

It is given that the set is $\{s,t,u,v\}$

The set of 3 permutations from the set $\{s,t,u,v\}$ is

$stu,\quad stv,\quad tuv,\quad suv$

$sut,\quad svt,\quad tvu,\quad svu$

$tus,\quad vts,\quad vut,\quad vus$

$tsu,\quad vst,\quad vtu,\quad vsu$

$uts,\quad tsv,\quad utv,\quad usv$

$ust,\quad tvs,\quad uvt,\quad uvs$

The number of 3 permutations from $\{s,t,u,v\}$ in $P(4,3)$

$$P(4,3)=\frac{4!}{(4-3)!}$$

$$=\frac{4!}{1!}$$

$$=4\times3\times2\times1$$

= 24

9.2

9.2.39

(a)

The objective is to find the number of ways to select three letters from *ALGORITHM*.

When the letters are arranged randomly in a row, the total number of arrangements is:

$9! = 362880.$

The number of ways of selecting 3 letters from the word *ALGORITHM* is the number of 3-permutations of a set of 9 elements.

This equals to:

$$\begin{aligned} P(9,3) &= \frac{9!}{(9-3)!} \\ &= \frac{9!}{6!} \\ &= \frac{9\cdot 8\cdot 7\cdot \cancel{6!}}{\cancel{6!}} \\ &= 504 \end{aligned}$$

Therefore, there are $\boxed{504}$ ways to select 3 letters of the word *ALGORITHM*.

Comment

Step 2 of 4

(b)

The objective is to find the number of ways to select six letters from *ALGORITHM*.

The number of ways of selecting 6 letters from the word *ALGORITHM* is the number of 6-permutations of a set of 9 elements.

This equals to:

$$\begin{aligned} P(9,6) &= \frac{9!}{(9-6)!} \\ &= \frac{9!}{3!} \\ &= \frac{9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot \cancel{3!}}{\cancel{3!}} \\ &= 60480 \end{aligned}$$

Therefore, there are $\boxed{60480}$ ways to select 6 letters of the word *ALGORITHM*.

9.2

(c)

The objective is to find the number of ways to select six letters from *ALGORITHM* if the first letter is A.

As the first position is fixed, the number of ways of selecting 6 letters from the word *ALGORITHM* is the number of 5-permutations of a set of 8 letters.

This equals to:

$$\begin{aligned} P(8,5) &= \frac{8!}{(8-5)!} \\ &= \frac{8!}{3!} \\ &= \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot \cancel{3!}}{\cancel{3!}} \\ &= 6720 \end{aligned}$$

Therefore, there are $\boxed{6720}$ ways to select 6 letters of the word *ALGORITHM* if the first letter is A.

Comment

Step 4 of 4

(d)

The objective is to find the number of ways to select six letters from *ALGORITHM* if the first two letters are OR.

As the first two positions are fixed, the number of ways of selecting 6 letters from the word *ALGORITHM* is the number of 4-permutations of a set of 7 letters.

This equals to:

$$\begin{aligned} P(7,4) &= \frac{7!}{(7-4)!} \\ &= \frac{7!}{3!} \\ &= \frac{7\cdot 6\cdot 5\cdot 4\cdot \cancel{3!}}{\cancel{3!}} \\ &= 840 \end{aligned}$$

Therefore, there are $\boxed{840}$ ways to select 6 letters of the word *ALGORITHM* if the first two letters are OR.

9.2

9.2.42

$$P(n+1,3)=\frac{(n+1)!}{(n+1-3)!}$$

$$=\frac{(n+1)!}{(n-2)!}$$

$$=\frac{(n+1)(n)(n-1)(n-2)!}{(n-2)!}$$

$$=(n+1)(n)(n-1)$$

$$=n(n^2-1)$$

$$=n^3-n$$

Comment

Step 2 of 4

$$P(n,3)=\frac{n!}{(n-3)!}$$

$$=\frac{n(n-1)(n-2)(n-3)!}{(n-3)!}$$

$$=n(n-1)(n-2)$$

$$=(n^2-n)(n-2)$$

$$=n^3-2n^2-n^2+2n$$

$$=n^3-3n^2+2n$$

9.2

Step 3 of 4

$$3P(n,2) = 3 \times \frac{n!}{(n-2)!}$$

$$= 3 \times \frac{n(n-1)(n-2)!}{(n-2)!}$$

$$= 3n(n-1)$$

$$= 3n^2 - 3n$$

Comment

Step 4 of 4

Now, consider that

$$P(n+1,3) - P(n,3)$$

$$= (n^3 - n) - (n^3 - 3n^2 + 2n)$$

$$= n^3 - n - n^3 + 3n^2 - 2n$$

$$= 3n^2 - 3n$$

$$\therefore P(n+1,3) - P(n,3) = 3P(n,2)$$

9.2

9.2.43

Step 1 of 1

Suppose n is an integer such that $n \geq 2$.

Use the following theorem:

The number of $r-$ permutations of a set of n elements is given as $P(n,r)=\frac{n!}{(n-r)!}$.

Rewrite the expression on the left hand side as follows:

$$P(n,n)=\frac{n!}{(n-n)!}$$

$$=\frac{n!}{0!}$$

$$=n!$$

Rewrite the expression on the right hand side as follows:

$$P(n,n-1)=\frac{n!}{(n-(n-1))!}$$

$$=\frac{n!}{(n-n+1)!}$$

$$=\frac{n!}{1!}$$

$$=n!$$

Thus, $P(n,n)=P(n,n-1)$.

9.3

9.3.9

(a)

The values of i varies from 1 to 4.

The values of j varies from 1 to i.

If $i=1$, then j will take only one value, that is 1.

Therefore, for $i=1$, the inner loop iterated once only.

If $i=2$, then j will take two values 1 and 2.

Therefore, for $i=2$, the inner loop iterated two times.

If $i=3$, then j will take three values 1, 2 and 3.

Therefore, for $i=3$, the inner loop iterated three times.

If $i=4$, then j will take three values 1, 2, 3 and 4.

Therefore, for $i=4$, the inner loop iterated four times.

Therefore, if the algorithm is implemented and run, then the inner loop runs

$\boxed{1+2+3+4=10}$ times.

(b)

Continuing from part (a), for $i=1,2,\ldots,n$, the inner loop is iterated

$\boxed{1+2+3+\ldots+n=\frac{n(n+1)}{2}}$ times.

9.3

9.3.13

(a)

The objective is to find the number of ways that eight people are seated beside each other.

If two people are sitting side by side, then there are 7 possible ways of selecting adjacent places.

Consider the two people as one person, and then the number of arrangements for 7 sitting is $P(7,7)$ ways.

As the two people can be arranged in $P(2,2)$ ways then the total number of arrangements is:

$$\begin{aligned} P(2,2)\times P(7,7) &= \frac{2!}{(2-2)!}\times\frac{7!}{(7-7)!} \\ &= 2!\times 7! \\ &= 2\times 5040 \\ &= 10,080 \end{aligned}$$

Therefore, the total number of arrangements is $\boxed{10,080}$ ways.

Comment

Step 2 of 2

(b)

The objective is to find the number of ways can eight be seated together in a row if two of the people do not sit beside each other.

The number of arrangements is: the total number of arrangements minus the total number of arrangements in which two people are together.

The total number of arrangements is $P(8,8)=\frac{8!}{(8-8)!}=8!=40320$ ways.

From part **(a)**, the number of arrangements in which two people are together is $10,080$ **ways.**

Thus, the number of ways is:

$$40,320-10,080=30,240$$

Therefore, the number of ways can eight be seated together in a row if two of the people do not sit beside each other is $\boxed{30,240}$ ways.

9.3

9.3.19

Step 1 of 1

It is given that a combination lock requires 3 selections of numbers from 1 through 39.

However, we are also given that no number can be used twice in a row, but the same number may occur in alternate positions.

Now, the first place can take any of the 39 numbers, but the next place can take any of the remaining 38 numbers, and now the third place can also take any of the 38 numbers,

i.e., excluding that number chosen in the second place.

$\therefore$ The total possibilities are $39 \times 38^2 = 56,316$

9.3

9.3.22

(a)

Total number of strings of length n over the set $\{a,b,c,d\}$ is 4^n.

The objective is to determine the number of strings of length n so that at least two adjacent characters are same.

This is equal to,

Total number of strings of length n

− Total number of strings such that no two adjacent characters are same.

Comment

Step 2 of 4

Total number of strings of length n over the set $\{a,b,c,d\}$ such that no two adjacent characters are same is obtained as follows:

Fill each of n places from the characters of the set $\{a,b,c,d\}$ to construct a string of n characters.

First place can be filled in 4 ways from the letters a,b,c and d.

Since no two adjacent characters are same, one cannot fill the second place with the character which is filled in the first place.

That is, one can fill the second place with the remaining three characters.

Hence, 2^{nd} place can be filled in 3 ways.

Similarly, third place cannot be filled with the character that occupied the second place but, can be filled with the character that is filled in the first place.

Continuing in this way, each place except the first place can be filled in 3 ways.

9.3

Therefore, by multiplication rule, the number of strings of length n such that no pair of adjacent characters is same = $4 \cdot \underbrace{3 \cdot 3 \cdot \ldots \cdot 3}_{n-1 \text{ times}} = 4 \cdot 3^{n-1}$

Therefore, total number of strings of length n such that at least two adjacent characters are same = total number of strings of length n − Total number of strings such that no two adjacent characters are same.

$= \boxed{4^n - 4 \cdot 3^{n-1}}$.

Comment

Step 4 of 4

(b)

Total number of strings of length 10 over the set $\{a,b,c,d\}$ is 4^{10}.

From part (a), the number of strings of length 10 such that at least one pair of adjacent characters that are same = $4^{10} - 4 \cdot 3^9$.

A string is chosen from the set of strings of length 10 over the set $\{a,b,c,d\}$.

Probability that the string contains at least one pair of adjacent characters is same

$$= \frac{\text{number of strings of length 10 so that at least two adjacent characters are same}}{\text{total number of strings of length 10}}$$

$$= \frac{4^{10} - 4 \cdot 3^9}{4^{10}}$$

$$= \boxed{1 - \left(\frac{3}{4}\right)^9}$$

9.3

9.3.23

Let A = the set of all integers from 1 through 1000 that are multiple of 4.
Let B = the set of all integers from 1 through 1000 that are multiple of 7.
Then
$A \cup B$ = the set of all integers from 1 through 1000 that are multiple of 7 or multiple of 4
and
$A \cap B$ = the set of all integers from 1 through 1000 that are multiple of both 7 and 4.
= the set of all integers from 1 through 1000 that are multiple of 28.

Comment

Step 2 of 5

Every integer from 1 through 1000 is a multiple of 4, each can be represented in the form $4k$, for some integer k from 1 through 250. Hence there are 250 multiples of 4 from 1 through 1,000, and so $N(A) = 250$.

Every integer from 1 through 1000 is a multiple of 7, each can be represented in the form $7k$, for some integer k from 1 through 142. Hence there are 142 multiples of 7 from 1 through 1,000, and so $N(B) = 142$.

Every integer from 1 through 1000 is a multiple of 28, each can be represented in the form $28k$, for some integer k from 1 through 35. Hence there are 35 multiples of 28 from 1 through 1,000, and so $N(A \cap B) = 35$.

Comment

Step 3 of 5

By the Inclusion/Exclusion Rule,

$$\begin{aligned} N(A \cup B) &= N(A) + N(B) - N(A \cap B) \\ &= 250 + 142 - 35 \\ &= 392 - 35 \\ &= 357 \end{aligned}$$

Therefore, the number of integers from 1 through 1000 that are multiple of 7 or multiple of 4 is $\boxed{357}$

9.3

Step 4 of 5

(b)

Since the number of integers from 1 through 1000 is 1000. And the number of integers from 1 through 1000 that are multiple of 7 or multiple of 4 is 357

Therefore, the probability that the integer is a multiple of 4 or a multiple of 7 is

$$\boxed{\frac{357}{1000} = 0.357}$$

Comment

Step 5 of 5

(c)

The number of integers from 1 through 1000 are neither multiples of 4 nor multiples of 7 is

$$\begin{aligned} N\left((A \cup B)^c\right) &= N(U - A \cup B) \\ &= N(U) - N(A \cup B) \\ &= 1000 - 357 \\ &= 643 \end{aligned}$$

9.3

9.3.34

Step 2 of 7

(a)

Since there are 50 subjects who were given the chance to use three drugs, the number of people who got relief from none of the drugs is,

$$50 - 41 = \boxed{9}.$$

Comment

Step 3 of 7

(b)

Use principal of inclusion and exclusion for three sets,

$$\begin{aligned} N(A \cup B \cup C) = N(A) + N(B) + N(C) - N(A \cap B) \\ - N(B \cap C) - N(A \cap C) + N(A \cap B \cap C) \\ 41 = 21 + 21 + 31 - 9 - 15 - 14 + N(A \cap B \cap C) \\ 41 = 35 + N(A \cap B \cap C) \\ N(A \cap B \cap C) = 6 \end{aligned}$$

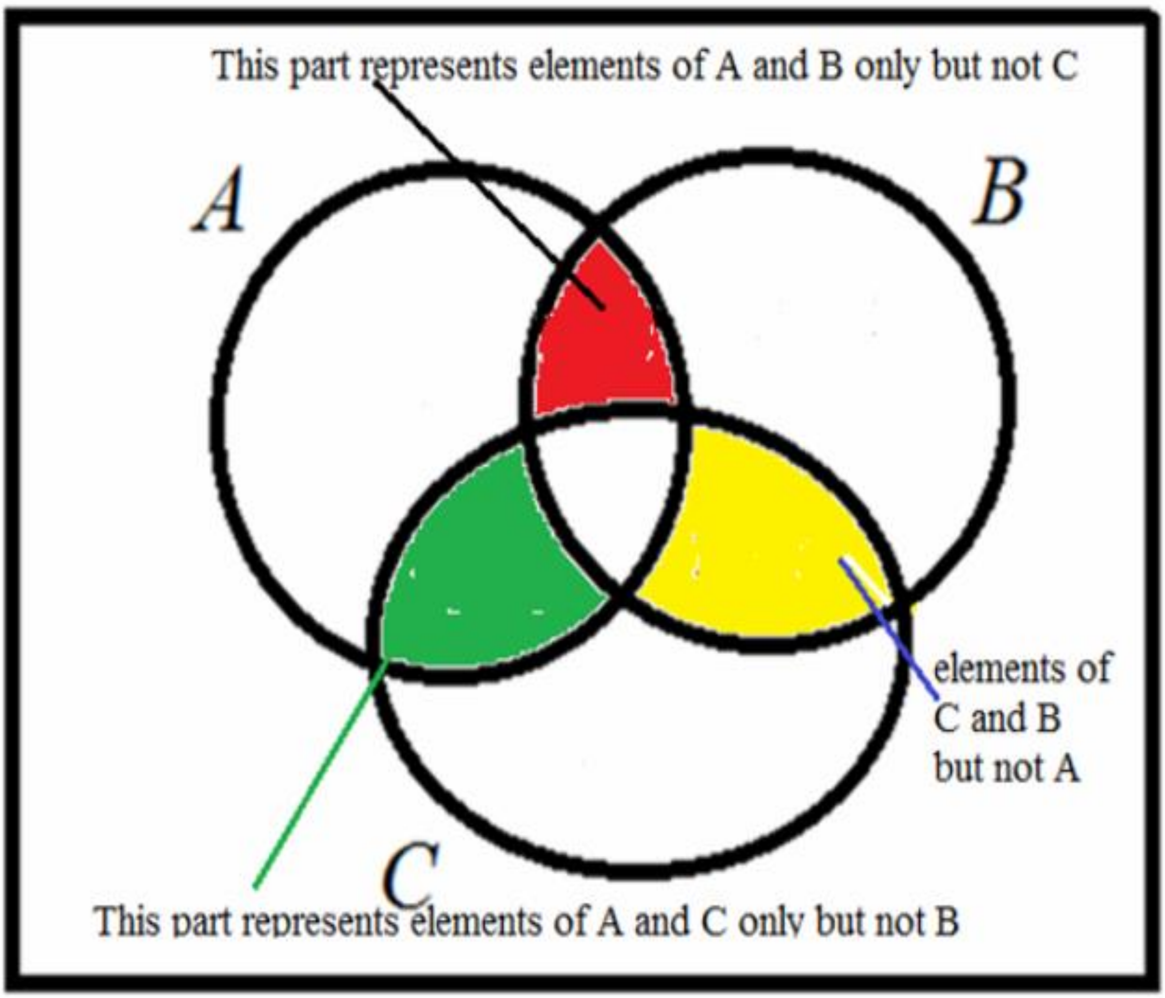
Therefore, number of people who got relief from all three drugs is $\boxed{6}$.

9.3

Step 4 of 7

(c)

Let A be the set of all subjects who got relief from drug A; and B be the set of all subjects who got relief from drug B; and C be the set of all subjects who got relief from drug C.



9.3

Step 5 of 7

Determine number of people who got relief from A and B only but not C.

This is equal to

Number of people who got relief from A and B

− Number of people who got relief from A, B and C.

Therefore, number of people who got relief from A and B only but not C is,

$$\begin{aligned} N(A \cap B) - N(A \cap B \cap C) &= 9 - 6 \\ &= 3 \end{aligned}$$

Similarly, number of people who got relief from B and C only but not A is equal to

Number of people who got relief from B and C

− Number of people who got relief from A, B and C

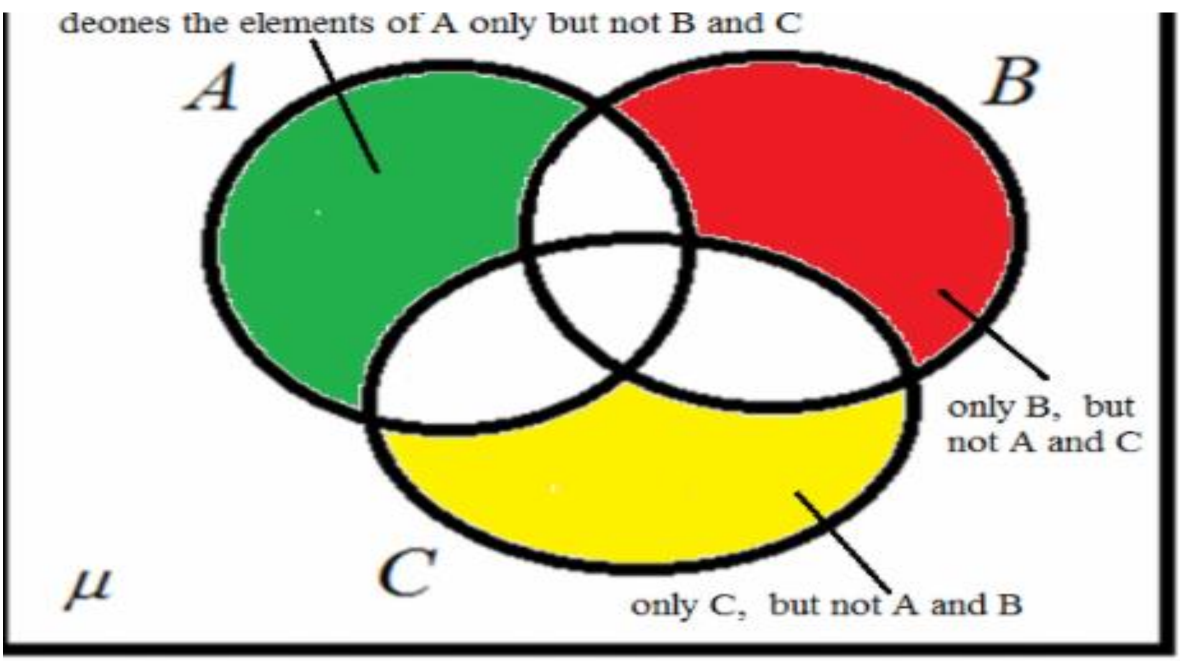
Therefore, number of people who got relief from B and C only but not A is,

$$\begin{aligned} N(B \cap C) - N(A \cap B \cap C) &= 15 - 6 \\ &= 9 \end{aligned}$$

Similarly, number of people who got relief from A and C only, but not B is equal to

$$\begin{aligned} N(A \cap C) - N(A \cap B \cap C) &= 14 - 6 \\ &= 8 \end{aligned}$$

9.3



termine number of people who got relief from A only, but not B and not C.

s is equal to,

$$(A)-N(A\cap B)-N(A\cap C)+N(A\cap B\cap C)=21-9-14+6$$
$$=4$$

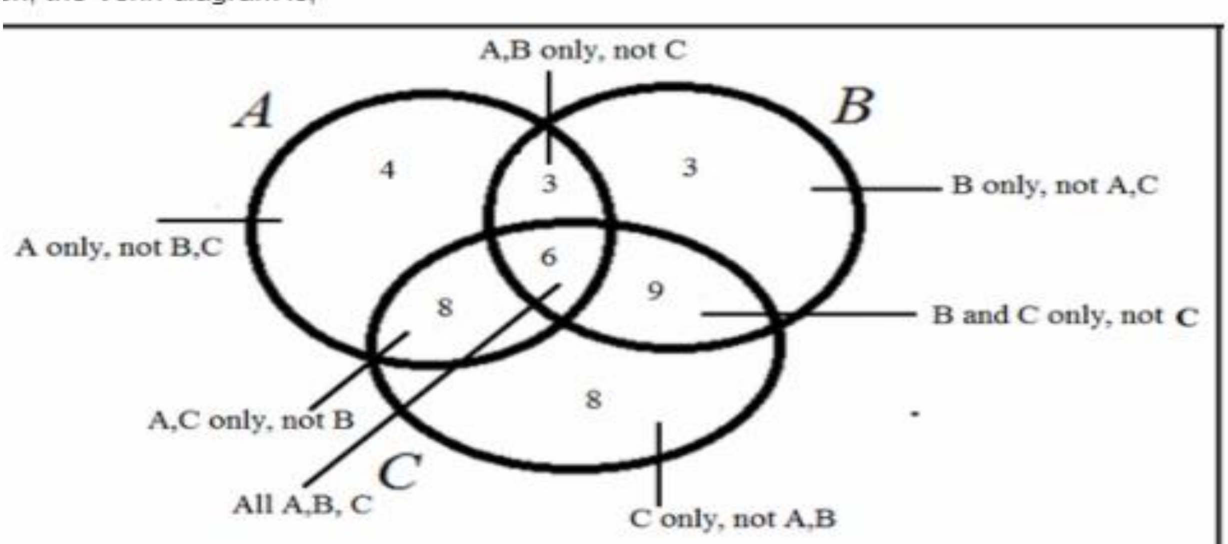
mber of people, who got relief from B only, and not from A and C is,

$$(B)-N(A\cap B)-N(B\cap C)+N(A\cap B\cap C)=21-9-15+6$$
$$=3$$

mber of people, who got relief from C only, and not from A and B is,

$$(C)-N(B\cap C)-N(A\cap C)+N(A\cap B\cap C)=31-14-15+6$$
$$=8$$

en, the Venn diagram is,



9.3

Step 7 of 7

(d)

The number of subjects who got relief only from A is,

$$N(A)-N(A\cap B)-N(A\cap C)+N(A\cap B\cap C)=21-9-14+6$$
$$=4$$

Therefore, number of subjects who got relief from only A is $\boxed{4}$.

9.3

9.3.35

Let *M* represent the set of married people in the given sample.

Let *Y* represent the set of people between 20 and 30 in the sample.

Let *F* represent the set of females in the sample.

Then, the number of people less than or equal to the size of the sample is represented by *MUYUF*.

According to the given data,

$N(M)=675$

$N(Y)=682$

$N(F)=684$

The number of people who are married and between 20 and 30 years old is given by $M \cap Y$

$\therefore N(M \cap Y)=195$

The number of people who are married and female is given by $M \cap F$

$\therefore N(M \cap F)=467$

The number of people who are female from 20 to 30 years old is $F \cap Y$

$\therefore N(F \cap Y)=318$

Comment

Step 2 of 2

The number of people who are married females from 20 to 30 years is 165

i.e., $N(M \cap Y \cap F)=165$

$$N(M \cup Y \cup F)=N(M)+N(Y)+N(F)-N(M \cap Y)-N(M \cap F) - N(Y \cap F)+N(M \cap Y \cap F)$$

$=675+682+684-195-467-318+165$

= 1226

However, it is given that the sample is of 1200 adults

i.e., $N(M \cup F \cup Y) \leq 1200$

Thus, the figures taken from the poll are inconsistent.

9.5

9.5.8

Students can choose 10 questions from 14 questions provided by instructor.

a.

The objective is t find the number of choices of 10 out of 14.

Let E be the event of choosing 10 questions out of 14 questions.

The number of ways selecting r number of items out of n is $\binom{n}{r}$.

The number of ways to select 10 questions from 14 is:

$$\begin{aligned} E &= \binom{14}{10} \\ &= \frac{14!}{10!(14-10)!} \\ &= \frac{14!}{10!\times 4!} \\ &= 1001 \end{aligned}$$

Thus, there are $\boxed{1001}$ choices to select 10 questions to answer.

Comment

Step 2 of 7

b.

If 6 questions require proof and 8 do not require proof.

(i)

The objective is to determine the number of groups of 10 questions contains 4 proof and 6 do not require proof.

The number of groups obtained of ten questions that contain 4 that require proof and six that do not are:

$$\begin{aligned} E_1 &= \binom{6}{4}\cdot\binom{8}{6} \\ &= \frac{6!}{4!(6-4)!}\cdot\frac{8!}{6!(8-6)!} \\ &= \frac{6!}{4!\cdot 2!}\cdot\frac{8!}{6!\cdot 2!} \\ &= 15\cdot 28 \\ &= 420 \end{aligned}$$

Thus, there are $\boxed{420 \text{ groups}}$ that contains ten questions with 4 that require proof and six that do not.

9.5

(ii)

The objective is to determine the number of groups of 10 questions that contains at least one require proof.

As there are 8 questions that do not require proof, all the possible groups are:

$$\begin{aligned} E_2 &= \text{Number of gropus contains at least one question that require proof} \\ &= S - \text{Number of groups that contains no question with proof} \\ &= 1001 - 0 \\ &= 1001 \end{aligned}$$

Thus, there are $\boxed{1001 \text{ groups}}$ of 10 questions that contain at least one question that requires proof.

Comment

Step 4 of 7

(iii)

The objective is to determine the number of groups of 10 questions that contains at most three that require proof.

If the group has 2 questions with proof and 8 do not require proof, then the number of possible groups is $\binom{6}{2}\cdot\binom{8}{8}$.

If the group has 3 questions with proof and 7 do not require proof, then the number of possible groups is $\binom{6}{3}\cdot\binom{8}{7}$.

The number of possible groups is:

$$\begin{aligned} E_3 &= \binom{6}{2}\cdot\binom{8}{8}+\binom{6}{3}\cdot\binom{8}{7} \\ &= 15\cdot 1 + 20\cdot 8 \\ &= 15 + 160 \\ &= 175 \end{aligned}$$

Thus, there will be $\boxed{175 \text{ groups}}$ of ten questions that contain at most three that require proof.

9.5

c.

The objective is to determine the different choices of 10 questions that at most one question of questions 1 and 2 may include among ten.

Student must answer at most one of questions 1 and 2.

That is he must not answer both.

So the required number of choices is:

$$E_4 = \begin{pmatrix} \text{total number} \\ \text{of choices} \end{pmatrix} - \begin{pmatrix} \text{number of choices which} \\ \text{include both 1 and 2} \end{pmatrix}.$$

From part **(a),** the total number of choices is 1001.

Comment

Step 6 of 7

Determine the number of choices which include both 1 and 2.

$$\begin{aligned} \binom{12}{8} &= \frac{12!}{8!(12-8)!} \\ &= \frac{12\cdot 11\cdot 10\cdot 9\cdot 8!}{8!4!} \\ &= 495 \end{aligned}$$

Then the number of groups is:

$$\begin{aligned} E_4 &= \begin{pmatrix} \text{total number} \\ \text{of choices} \end{pmatrix} - \begin{pmatrix} \text{number of choices which} \\ \text{include both 1 and 2} \end{pmatrix} \\ &= 1001 - 495 \\ &= 506 \end{aligned}$$

Thus, the required number of groups is $\boxed{506}$.

9.5

Step 7 of 7

d.

The objective is to determine the different choices of 10 questions that either both 1 and 2 are included or neither.

If both the questions 1 and 2 are neglected, then the number of possible groups is $\binom{2}{0}\cdot\binom{12}{10}$.

If questions 1 and 2 are included, then the number of possible groups is $\binom{2}{2}\cdot\binom{12}{8}$.

Then the number of groups is:

$$\begin{aligned} E_5 &= \binom{2}{0}\cdot\binom{12}{10}+\binom{2}{2}\cdot\binom{12}{8} \\ &= 1\cdot 66 + 1\cdot 495 \\ &= 66+495 \\ &= 561 \end{aligned}$$

Thus, the required groups will be $\boxed{561}$.

9.5

9.5.15

Step 1 of 4

(a) The number of even integers in the set $\{1,2,3...100\}$ is $\frac{100}{2}=50$

Comment

Step 2 of 4

(b) The number of odd integers in the set $\{1,2,3...100\}$ is $\frac{100}{2}=50$

Comment

9.5

(c) The sum of 2 integers can be even if both are even or if both are odd

The two even numbers that can be selected from the 50 even numbers is $\binom{50}{2}$ ways.

The two odd integers that can be selected from the 50 odd integers is $\binom{50}{2}$ ways.

$\therefore$ The total possibilities are

$$=\binom{50}{2}+\binom{50}{2}$$

$$=2\times\binom{50}{2}$$

$$=2\times\frac{50!}{2!\times 48!}$$

$$=50\times 49$$

$$=2450$$

Comment

Step **4** of 4

(d) The sum of two integers can be odd if one of them is odd and the other is even.

One odd integer can be selected from the 50 odd integers in $\binom{50}{1}$ ways.

Similarly, one odd integer can be selected from 50 even integers in $\binom{50}{1}$ ways.

$\therefore$ The total possibilities are

$$=\binom{50}{1}\times\binom{50}{1}$$

$$=50\times 50$$

$$=2500$$

9.5

9.5.17

Step 1 of 5

It is given that 10 points are arranged in a plane and labeled as

A, B, C, D, E, F, G, H, I, J.

Comment

Step 2 of 5

(a) The number of straight lines from these 10 points is obtained by selecting 2 points from the 10 points. It is given by $\binom{10}{2}$

$$\binom{10}{2} = \frac{10!}{2! \times 8!}$$

$$= \frac{10 \times 9}{2}$$

$$= \boxed{45}$$

Comment

Step 3 of 5

(b) The number of straight lines which do not pass through *A* is the number of straight lines formed by the other 9 points

$$\therefore \binom{9}{2} = \frac{9!}{2! \times 7!}$$

$$= \frac{9 \times 8}{2}$$

$$= \boxed{36}$$

9.5

Step 4 of 5

(c) The number of triangles that can be formed from these 10 points is all the selections of 3 points from the 10 points

$\therefore \binom{10}{3}$ ways

$$\binom{10}{3} = \frac{10!}{3! \times 7!}$$

$$= \frac{10 \times 9 \times 8 \times 7!}{3! \times 7!}$$

$$= \frac{10 \times 9 \times 8}{3 \times 2 \times 1}$$

$$= \boxed{120}$$

Comment

Step 5 of 5

(d) The number of triangles which do not have *A* as the vertex is the number of triangles formed from the remaining 9 points.

This can be done in $\binom{9}{3}$ ways.

$$\binom{9}{3} = \frac{9!}{3! \times 6!}$$

$$= \frac{9 \times 8 \times 7 \times 6!}{3! \times 6!}$$

$$= \frac{9 \times 8 \times 7}{3 \times 2 \times 1} = 3 \times 4 \times 7$$

$$= \boxed{84}$$

9.5

9.5.19

(a)

The given word is $HULLABALOO$.

There are 10 positions, and have to be filled with 1 H, 1 U, 3L's, 2A's 1B and 2 O's.

Step 1: Choose 3 positions for 3L's

This can be done in $\binom{10}{3}$ ways.

Step 2: Choose 2 positions for 2A's from the remaining 7 positions.

This can be done in $\binom{7}{2}$ ways.

Step 3: Choose 2 positions for 2 O's from the remaining 5 positions.

This can be done in $\binom{5}{2}$ ways

Step 4: Chose 1 position for H from the remaining 3 positions.

This can be done in $\binom{3}{1}$ ways.

Step 5: Choose 1 position for U from the remaining 2 positions.

This can be done in $\binom{2}{1}$ ways.

And B can be placed in the remaining position.

Comment

Step 2 of 4

Therefore, total number of ways the letters from the word " $HULLABALOO$ " are arranged is,

$$\binom{10}{3}\binom{7}{2}\binom{5}{2}\binom{3}{1}\binom{2}{1}\binom{1}{1} = \frac{10!}{3!\times 7!}\times\frac{7!}{2!\times 5!}\times\frac{5!}{2!\times 3!}\times\frac{3!}{1!\times 2!}\times\frac{2!}{1!\times 1!}\times\frac{1!}{0!\times 1!}$$

$$= \frac{10!}{3!\,2!\,2!}$$

$$= 151{,}200$$

9.5

(b)

The objective is to find the number of distinguishable orderings of the letters of the word HULLABALOO, that begins with U and ends with L.

$\boxed{U}$ _ _ _ _ _ _ _ _ $\boxed{L}$

Every ordering of the letters should have U in the first place and L in the last place. So, there are 8 positions and 8 letters; 2L's, 2A's, 2O's, 1H, 1B are left.

Arrange 2L's in 8 positions. This is done in $\binom{8}{2}$ ways.

Arrange 2A's in the remaining 6 positions. This is done in $\binom{6}{2}$ ways.

Arrange 2O's in the remaining 4 positions. This is done in $\binom{4}{2}$ ways.

Arrange H and B in the remaining 2 positions. This is done in $2!$ ways.

Therefore, total number of distinguishable orderings of the letters of the word HULLABALOO, that are begins with U and ends with L is,

$$\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{1}\binom{1}{1} = \frac{8!}{2!\times 2!\times 2!}$$
$$= \boxed{5,040}$$

Comment

Step 4 of 4

(c)

The objective is to find the number of distinguishable orderings of the letters of the word HULLABALOO, in which H and U are next to each other in order.

Assume the two letters H U as one letter next to each other in order.

The remaining 8 letters are 3L's, 2A's, 2O's, 1B.

Therefore, total number of letters is 9, including H U as one letter.

Number of ways arranging these 9 letters is,

$$\binom{9}{3}\binom{6}{2}\binom{4}{2}2! = \frac{9!}{3!\times 6!}\times\frac{6!}{2!\times 4!}\times\frac{4!}{2!\times 2!}\times 2!$$
$$= \frac{9!}{3!\,2!\,2!}$$
$$= 15,120$$

Therefore, the total number of distinguishable orderings of the letters of the word HULLABALOO, in which H and U are next to each other in order is, $\boxed{15,120}$.

9.5

9.5.21

Step 1 of 3

Let us consider the Morse code, in which symbols are represented by variable-length sequences of dots and dashes.

Let us consider the case of 1 symbol (either dash or dot).

Total patterns from with 1 symbol is 2 (? and ·)

Comment

Step 2 of 3

Let us consider the case of 2 symbols (either dash or dot).

Possible patterns in this case are −− , −· , ·− , and ··

Total patterns from with 2 symbol is $2^2 = 4$

Comment

Step 3 of 3

Similarly, if there are 3 symbols, total patterns from are $2^3 = 8$ and so on till 7 symbols.

Therefore, the total number of patterns formed with seven or fewer dots and dashes is as follows

$2^1 + 2^2 + 2^2 ... + 2^7 = 2(2^7 - 1)$ (Using summation of geometric progression series)

$$
\begin{aligned}
&= 2^8 - 2 \\
&= 256 - 2 \\
&= \boxed{254}
\end{aligned}
$$

9.5

9.5.23

Step 1 of 1

The objective is to determine number of different paths on a 8×8 chessboard so that a rook moves from the bottom left square of chess board to the top right square of chess board. If the rook is allowed to move any number of squares, either horizontally or vertically, all moves are to the right or upward.

The rook starts at the bottom left square of chess board.

The rook can move either to right or upward.

If the rook moves to right, then denotes it with R.

If the rook moves to upward, then denotes it with U.

The rook requires 7 rights and 7 upwards to reach the top right square from the bottom left square of chess board.

So, number of different paths is equal to the number of ordering of 7R's and 7U's.

The number of differing ordering of 7R's and 7U's is,

$$\frac{14!}{7!\times 7!} = 3,432.$$

Therefore, the number of differing paths for the rook, to reach the top right square from the bottom left square of chess board is, $\boxed{3,432}$.

9.5

9.5.25

(a) We have to find the number of one-to-one functions from set with three elements to the set with four elements.

So there are four chances to send first element in domain to co-domain

Since the function is one-to-one, there are three choices to send second element and there are two choices to send the third element

So the number of one-to-one functions is

$$= 4 \cdot 3 \cdot 2$$
$$= 24$$

Comment

Step 2 of 5

(b) We have to find the number of one-to-one functions from set with three elements to the set with two elements.

Since the domain has more elements than co-domain so we cannot find a one-to-one function between these sets.

Comment

Step 3 of 5

(c) We have to find the number of one-to-one functions from set with three elements to the set with three elements.

So there are three chances to send first element in domain to co-domain

Since the function is one-to-one, there are two choices to send second element and one choice to send the third element

So the number of one-to-one functions is

$$= 3 \cdot 2 \cdot 1$$
$$= 6$$

9.5

Step 4 of 5

(d) We have to find the number of one-to-one functions from set with three elements to the set with five elements.

So there are five chances to send first element in domain to co-domain

Since the function is one-to-one, there are four choices to send second element and three choices to send the third element

So the number of one-to-one functions is

$$= 5 \cdot 4 \cdot 3$$
$$= 60$$

Comment

Step 5 of 5

(e) We have to find the number of one-to-one functions from set with m elements to the set with n elements.

Since $m \le n$ so there exists one-one function between the sets

And there are $(n-1)$ chances to send first element in domain to co-domain

Since the function is one-to-one, there are $(n-2)$ choices to send second element and so on.

So the number of one-to-one functions is

$$n(n-1)(n-2).....(n-m+1) = P(n,m) \quad (m \le n)$$

9.5

9.5.27

Step 1 of 4

Let A be a set with 8 elements.

(a)

The objective is to determine the number of relations on set A.

A relation A is a subset of $A \times A$.

As $A \times A$ has n^2 elements, there are 2^{n^2} subsets.

Thus, there are 2^{n^2} relations on a set with n elements.

Here A has 8 elements, so substitute $n = 8$ in 2^{n^2}.

Therefore, there are $2^{8^2} = \boxed{2^{64}}$ relations on set A.

Comment

Step 2 of 4

(b)

The objective is to determine the number of reflexive relations on A.

For reflexive relations there is only one way in set A, for remaining $n^2 - n$ elements there are 2 choices for each.

So, total number of reflexive relations for n elements in set A is:

$$1 \times 2^{n^2 - n} = 2^{n^2 - n} = 2^{n(n-1)}.$$

Here A has 8 elements, so substitute $n = 8$ in the formula.

Therefore, there are $2^{8(8-1)} = \boxed{2^{56}}$ reflexive relations on A.

9.5

(c)

The objective is to determine the number of symmetric relations on A.

For (i,i) there are 2 choices for each either it can include in a relation or it cannot include in relation.

So, 2^n ways.

By the definition of symmetric relations, there are pairs of $(1,2),(2,1)$ and $(2,3),(3,2)$, so on.

So, for n^2-n elements, only half of the values are selected.

The number of ways for n^2-n elements is $2^{\frac{n^2-n}{2}}$.

By multiplication rule, the total number of ways is $2^n \times 2^{\frac{n^2-n}{2}} = 2^{\frac{2n+n^2-n}{2}} = 2^{\frac{n(n+1)}{2}}$.

Here A has 8 elements, so substitute $n=8$ in the formula.

Therefore, there are $2^{\frac{8(8+1)}{2}} = \boxed{2^{36}}$ symmetric relations on A.

Comment

Step 4 of 4

(d)

The objective is to determine the number of both reflexive and symmetric relations on A.

For (i,i) there is only one choice that is they all must be included in the relation.

For remaining n^2-n elements, only half of the values are selected.

The number of ways for n^2-n elements is $2^{\frac{n^2-n}{2}}$.

By multiplication rule, the total number of ways is $1 \times 2^{\frac{n^2-n}{2}} = 2^{\frac{n(n-1)}{2}}$.

Here A has 8 elements, so substitute $n=8$ in the formula.

Therefore, there are $2^{\frac{8(8-1)}{2}} = \boxed{2^{28}}$ both reflexive and symmetric relations on A.

9.5

(c)

The objective is to determine the number of symmetric relations on A.

For (i,i) there are 2 choices for each either it can include in a relation or it cannot include in relation.

So, 2^n ways.

By the definition of symmetric relations, there are pairs of $(1,2),(2,1)$ and $(2,3),(3,2)$, so on.

So, for n^2-n elements, only half of the values are selected.

The number of ways for n^2-n elements is $2^{\frac{n^2-n}{2}}$.

By multiplication rule, the total number of ways is $2^n \times 2^{\frac{n^2-n}{2}} = 2^{\frac{2n+n^2-n}{2}} = 2^{\frac{n(n+1)}{2}}$.

Here A has 8 elements, so substitute $n=8$ in the formula.

Therefore, there are $2^{\frac{8(8+1)}{2}} = \boxed{2^{36}}$ symmetric relations on A.

Comment

Step 4 of 4

(d)

The objective is to determine the number of both reflexive and symmetric relations on A.

For (i,i) there is only one choice that is they all must be included in the relation.

For remaining n^2-n elements, only half of the values are selected.

The number of ways for n^2-n elements is $2^{\frac{n^2-n}{2}}$.

By multiplication rule, the total number of ways is $1 \times 2^{\frac{n^2-n}{2}} = 2^{\frac{n(n-1)}{2}}$.

Here A has 8 elements, so substitute $n=8$ in the formula.

Therefore, there are $2^{\frac{8(8-1)}{2}} = \boxed{2^{28}}$ both reflexive and symmetric relations on A.

9.5

9.5.28

Step 1 of 4

Consider a student council consists of three freshmen, four sophomores, four juniors and five seniors.

The objective is to find number of committees of eight members can be formed from the above council such that the committee should contain at least one member from each class.

Comment

Step 2 of 4

The number of ways to choose a subset of r elements from a set of n elements is:

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

The other notation is as follows:

$$C(n,r)=\binom{n}{r}.$$

9.5

Since the committee must contain at least one member from each class, first choose one member from each class in the following way:

One member from three freshmen can be selected in $\binom{3}{1}$ ways.

One member from four sophomores can be selected in $\binom{4}{1}$ ways.

One member from four juniors can be selected in $\binom{4}{1}$ ways.

One member from five seniors can be selected in $\binom{5}{1}$ ways.

Hence, the first four members of the committee can be chosen in $\binom{3}{1}\binom{4}{1}\binom{4}{1}\binom{5}{1}$ ways.

That is,

$$\begin{aligned}\binom{3}{1}\binom{4}{1}\binom{4}{1}\binom{5}{1} &= \left[\frac{3!}{1!(3-1)!}\right]\left[\frac{4!}{1!(4-1)!}\right]\left[\frac{4!}{1!(4-1)!}\right]\left[\frac{5!}{1!(5-1)!}\right] \\ &= \frac{3\times \cancel{2!}}{\cancel{2!}}\times\frac{4\times \cancel{3!}}{\cancel{3!}}\times\frac{4\times \cancel{3!}}{\cancel{3!}}\times\frac{5\times \cancel{4!}}{\cancel{4!}}\end{aligned}$$

$$\binom{3}{1}\binom{4}{1}\binom{4}{1}\binom{5}{1} = 3\times4\times4\times5$$

$$\binom{3}{1}\binom{4}{1}\binom{4}{1}\binom{5}{1} = 240$$

Comment

Step 4 of 4

After choosing the four members of the committee in the above described manner, there are two freshmen, three sophomores, three juniors and four seniors left in the student council.

And the remaining four members of the committee can be chosen randomly from two freshmen, three sophomores, three juniors and four seniors $(2+3+3+4=12)$ that are left in the student council.

Use inclusion/exclusion rule to find the total number of committees.

Thais can be done in

$$\begin{aligned}\binom{12}{4} &= \frac{12!}{(12-4)!(4!)} \\ &= \frac{\cancel{12}\times11\times10\times9\times\cancel{8!}}{\cancel{8!}\times\cancel{4\times3}\times2\times1} \\ &= \frac{990}{2} \\ &= 495 \text{ ways.}\end{aligned}$$

Therefore, the total number of committees that can be formed such that the committee should contain at least one member from each class is,

$$240\times495=\boxed{1,18,800}\text{ ways.}$$

9.6

9.6.1

The number of *r* combinations with repetition allowed that can be selected from a set of *n* elements is $\binom{r+n-1}{r}$

Comment

Step 2 of 3

(a) Now, we have to find the 5 combinations from a set of 3 elements, when repetition is allowed from the formula $\binom{r+n-1}{r}$. We have $\binom{5+3-1}{5}$ ways.

$$\binom{5+3-1}{5} = \binom{7}{5} = \frac{7!}{5!(7-5!)}$$

$$= \frac{7!}{5!\times 2!}$$

$$= \frac{7\times 6}{2}$$

$$= 21$$

Comment

Step 3 of 3

(b) The list of all the 5 combinations from $\{1,2,3\}$ when repetition is allowed is

$$\left\{\begin{array}{l} \{11111\}\{11112\}\{11113\}\{11122\}\{11123\}\{11133\} \\ \{22222\}\{22223\}\{22221\}\{22211\}\{22213\}\{22233\} \\ \{33333\}\{33331\}\{33332\}\{33311\}\{33322\}\{33312\} \\ \{12233\}\{11233\}\{11223\} \end{array}\right\}$$

9.6

9.6.3

Six variety of pastry including eclairs are produced in a bakery. There are 20 minimum pastries of each kind.

Comment

Step 2 of 4

(a)

The main objective is to find the various selections of twenty pastries which are there.

Since there are 20 minimum pastries of each kind, $n = 20$.

Also, six variety of pastry are present. So, $k = 6$

The number of ways to select m things from n things where repetition is allowed is $\binom{n+m-1}{n}$.

$$\begin{aligned}\text{Total selections} &= \binom{20+6-1}{20} \\ &= \binom{25}{20} \\ &= 53130\end{aligned}$$

Thus, the different selections of twenty pastries which are there is $53,130$.

9.6

(b)

The main objective is to find the various selections of twenty pastries where a minimum of three eclairs is there.

Since a minimum of three eclairs is present, $n = 17$.

Also, six variety of pastry are present. $k = 6$

The number of ways to select m things from n things where repetition is allowed is $\binom{n+m-1}{n}$.

$$\text{Total selections} = \binom{17+6-1}{17}$$
$$= \binom{22}{17}$$
$$= 26334$$

Thus, the different selections of twenty pastries where a minimum of three eclairs is there is $26,334$.

Comment

Step 4 of 4

(c)

The main objective is to find the various selections of twenty pastries where a maximum of two eclairs is there.

This is equivalent to calculate the total different selections of twenty pastries and then subtracting from different selections of twenty pastries where a minimum of three eclairs is there.

Total ways $= 53130 - 26,334 = 26796$

Thus, the different selections of twenty pastries where a maximum of two eclairs is there. is $26,796$.

9.6

9.6.4

Step 1 of 4

Eight variety of battery including $A76$ are in stock in a shop. There are minimum 30 battery for each kind.

Comment

Step 2 of 4

(a)

The main objective is to find the total number of inventory of 30 batteries which could be distributed among eight different types.

Since there are minimum 30 battery for each kind., $n = 30$.

$k = 8$

The number of ways to select m things from n things where repetition is allowed is $\binom{n+m-1}{n}$

.

$$\text{Total selections} = \binom{30+8-1}{30} = \binom{37}{30} = 10295472$$

Thus, the total number of inventory of 30 batteries which could be distributed among eight different types is $10{,}295{,}472$.

9.6

(b)

The main objective is to find the total number of inventory of 30 batteries which could be distributed among eight different types if at least four A76 battery are included.

Since at least four A76 battery are included., $n = 30 - 4 = 26$.

$k = 8$

The number of ways to select m things from n things where repetition is allowed is $\binom{n+m-1}{n}$.

$$\text{Total selections} = \binom{26+8-1}{26} = \binom{33}{26} = 4272048$$

Thus, the total number of inventory of 30 batteries which could be distributed among eight different types if at least four A76 battery are included is $4{,}272{,}048$.

Comment

Step 4 of 4

(c)

The main objective is to find the total number of inventory of 30 batteries which could be distributed among eight different types if at most three A76 battery are included.

This is equivalent to calculating the total inventory of 30 batteries and then subtracting from the total number of inventory of 30 batteries which could be distributed among eight different types if at least four A76 battery are included.

Total ways $= 10{,}295{,}472 - 4{,}272{,}048 = 6023424$

Thus, the total number of inventory of 30 batteries which could be distributed among eight different types if at most three A76 battery are included is $6{,}023{,}424$.

9.6

9.6.8

Step 1 of 2

Construct a trace table for the values of m, k, j and i for which the statement in the body of the inner most loop are executed.

Observe that *i* goes from 1 to j. It is always the case that $i \le j$. Similarly, because j goes from 1 to k, it is always the case that $j \le k$ and similarly $k \le m.$

Observe that there is one iteration of the inner most loop for each column and there is one column of the table for each quadruples of integers (i, j, k, m) with $1 \le i \le j \le k \le m \le n$.

Comment

Step 2 of 2

The total number of quadruples is same as 4-combinations with repetition allowed that can be formed from a set of n elements. This is given by,

$$\binom{4+n-1}{4} = \binom{n+3}{4}$$
$$= \frac{n(n+1)(n+2)(n+3)}{24}$$

Hence, the total number of quadruples is $\boxed{\frac{n(n+1)(n+2)(n+3)}{24}}$.

9.6

9.6.13

The objective is to find the number of solutions to the equation $y_1 + y_2 + y_3 + y_4 = 30$, for each $y_i \geq 2$.

Let $x_i = y_i - 2$ for each $i = 1,2,3,4$.

Then, for each $x_i \geq 0$.

$$\begin{aligned} y_1 + y_2 + y_3 + y_4 &= 30 \\ (x_1 + 2) + (x_2 + 2) + (x_3 + 2) + (x_4 + 2) &= 30 \\ x_1 + x_2 + x_3 + x_4 + 8 &= 30 \\ x_1 + x_2 + x_3 + x_4 &= 22 \end{aligned}$$

Comment

Step 2 of 2

Assume that the number 22 is divided into twenty two individual units, and the variables x_1, x_2, x_3, x_4 are four categories into which these units are placed.

The number of units in each category x_i indicates that the value of x_i is a solution of the equation.

Distribute the remaining 22 units among the 4 categories.

The total number of ways is:

$$\begin{aligned} \binom{22+4-1}{22} &= \binom{25}{22} \\ &= \frac{25!}{22!(25-22)!} \\ &= \frac{25!}{22!3!} \\ &= \frac{25 \cdot 24 \cdot 23 \cdot \cancel{22!}}{\cancel{22!} \cdot 3 \cdot 2 \cdot 1} \\ &= 2300 \end{aligned}$$

Therefore, there are $\boxed{2300}$ integral solutions of the equation where each integer in the solution is at least 2.

9.6

(a)

The main objective of is to calculate the different ways in which 15 can of drink can be selected if the shop has only six can of lemonade but at least 15 cans of each other four types of drinks.

Result Used:

The number of ways to select *m* things from *n* things where repetition is allowed is $\binom{n+m-1}{n}$.

Comment

Step 3 of 6

This is equivalent to find the total selection of cans of any one of five type if there are at least 15 types of each type and then subtracting from total selections where there are at least seven cans of lemonade.

For total selections where there are at least seven cans of lemonade, assume the following.

$n = 15 - 7 = 8$

$k = 5$

In this case, total ways is $\binom{8+5-1}{8} = 495$

From example 9.6.2, total selection of cans of any one of five type if there are at least 15 types of each type is 3876.

$$\begin{aligned} \text{Total ways} &= 3876 - 495 \\ &= 3381 \end{aligned}$$

Thus, the different ways in which 15 can of drink can be selected if the shop has only six can of lemonade but at least 15 cans of each other four types of drinks is $3,381$.

9.6

(b)

The main objective of is to calculate the different ways in which 15 can of drink can be selected if the shop has only five cans of beer and six can of lemonade but at least 15 cans of each other three types of drinks.

Result Used:

The number of ways to select *m* things from *n* things where repetition is allowed is $\binom{n+m-1}{n}$.

Comment

Step 5 of 6

Assume that $r_{\le 5}$ be the set of selections having at most 5 cans of beer and $l_{\le 6}$ be the set of sections having at most six cans of lemonade. The value of $n(r_{\le 5} \cap l_{\le 6})$ is the required answer.

$$n(r_{\le 5} \cap l_{\le 6}) = n(T) - n(r_{\ge 6} \cup l_{\ge 7})$$

By the inclusion/exclusion rule,

$$n(r_{\ge 6} \cup l_{\ge 7}) = n(r_{\ge 6}) + n(l_{\ge 7}) - n(r_{\ge 6} \cap l_{\ge 7})$$

From part a, $n(l_{\ge 7}) = 495$

From part b of Example 9.6.2, $n(r_{\ge 6}) = 715$

Calculate the value of $n(r_{\ge 6} \cap l_{\ge 7})$ as follows:

In this case, at most 6 cans of beer and at most 7 cans of lemonade are selected. So, a maximum of 2 more can of drinks are selected from three types to make up the total of 15 cans.

$$\begin{aligned} n(r_{\ge 6} \cap l_{\ge 7}) &= \binom{2+3-1}{2} + \binom{1+3-1}{1} \\ &= \binom{4}{2} + \binom{3}{1} \\ &= 6+3 \\ &= 9 \end{aligned}$$

Substitute these values in $n(r_{\ge 6} \cup l_{\ge 7}) = n(r_{\ge 6}) + n(l_{\ge 7}) - n(r_{\ge 6} \cap l_{\ge 7})$ to get,

$$\begin{aligned} n(r_{\ge 6} \cup l_{\ge 7}) &= 715 + 495 - 9 \\ &= 1201 \end{aligned}$$

9.6

Step 6 of 6

Now substitute these values in $n(r_{\le 5} \cap l_{\le 6}) = n(T) - n(r_{\ge 6} \cup l_{\ge 7})$,

$$n(r_{\le 5} \cap l_{\le 6}) = 3876 - 1201$$
$$= 2{,}675$$

Thus, the different ways in which 15 can of drink can be selected if the shop has only five cans of beer and six can of lemonade but at least 15 cans of each other three types of drinks

is $2{,}675$.

Comment

9.6

9.6.18

(a)

There are 10 eclairs. There are a minimum of 20 pastries of each of the other kind.

The main objective of is to calculate total differentselections of twenty pastries.

Comment

Step 2 of 8

Result Used:

The number of ways to select m things from n things where repetition is allowed is $\binom{n+m-1}{n}$.

Comment

Step 3 of 8

Let S be the set of selecting 20 pastries where there are a minimum of 20 of each type. Let $e_{\le 10}$ and $e_{\ge 11}$ be the set containing maximum 10 and 11 minimum eclairsrespectively.

From exercise 3, $n(S) = 53{,}130$.

Clearly, $S = e_{\le 10} \cup e_{\ge 11}$ and $e_{\le 10} \cap e_{\ge 11} = \phi$. This implies $n(S) = n(e_{\le 10}) + n(e_{\ge 11})$.

Compute $n(e_{\ge 11})$ as follows:

In this case,

$n = 9$ and $m = 6$

In this case, total ways is $\binom{9+6-1}{9} = \binom{14}{9} = 2002$.

Since $n(S) = n(e_{\le 10}) + n(e_{\ge 11})$,

$$\begin{aligned} n(e_{\le 10}) &= n(S) - n(e_{\ge 11}) \\ &= 53{,}130 - 2002 \\ &= 51{,}128 \end{aligned}$$

Thus, the total different selections of twenty pastries is $51{,}128$.

9.6

Step 4 of 8

(b)

There are 10 eclairs and 8 napoleon slices. There are a minimum of 20 pastries of each of the other kind.

The main objective of is to calculate total different selections of twenty pastries.

Comment

Step 5 of 8

Result Used:

The number of ways to select *m* things from *n* things where repetition is allowed is $\binom{n+m-1}{n}$.

Comment

Step 6 of 8

Let *S* be the set of selecting 20 pastries where there are a minimum of 20 of each type. Let $p_{\le 8}$ and $p_{\ge 9}$ be the set containing maximum 8 and 9 minimum napoleon slices respectively.

Clearly, $S = p_{\le 8} \cup p_{\ge 9}$ and $p_{\le 8} \cap p_{\ge 9} = \phi$. This implies $n(S) = n(p_{\le 8}) + n(p_{\ge 9})$.

Compute $n(p_{\ge 9})$ as follows:

In this case,

$n = 10 - 9 = 11$ and $m = 6$.

In this case, total ways is $\binom{11+6-1}{11} = \binom{16}{11} = 4,368$.

Since $n(S) = n(p_{\le 8}) + n(p_{\ge 9})$.

$$\begin{aligned} n(p_{\le 8}) &= n(S) - n(p_{\ge 9}) \\ &= 53,130 - 4386 \\ &= 48,744 \end{aligned}$$

9.6

Step 7 of 8

Let $e_{\geq 11}$ and $p_{\geq 9}$ be the set containing minimum 11 eclairs and 9napoleon slices respectively.

Clearly, $n(e_{\geq 11} \cup p_{\geq 9}) = n(e_{\geq 11}) + n(p_{\geq 9}) - n(e_{\geq 11} \cap p_{\geq 9})$.

$$\begin{aligned} n(e_{\geq 11} \cup p_{\geq 9}) &= n(e_{\geq 11}) + n(p_{\geq 9}) - n(e_{\geq 11} \cap p_{\geq 9}) \\ &= 2002 + 4386 - 1 \qquad [\because n(e_{\geq 11} \cap p_{\geq 9}) = 1] \\ &= 6,387 \end{aligned}$$

Comment

Step 8 of 8

Since $n(S) = n(e_{\leq 10} \cup p_{\leq 8}) + n(e_{\geq 11} \cup p_{\geq 9})$,

$$\begin{aligned} n(e_{\leq 10} \cup p_{\leq 8}) &= n(S) - n(e_{\geq 11} \cup p_{\geq 9}) \\ &= 53,130 - 6,387 \\ &= 46,743 \end{aligned}$$

Thus, the total different selections of twenty pastries is $46,743$.